A System of Simple Consistent Definitions for Power Quantities

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Power Measurements

Today the measurement of power quantities is done almost exclusively by digital sampling based algorithms. When one has a stream of data based on constant frequency sampling one can select a set of definitions for Active Power, Apparent Power and Reactive Power which are easily implemented in modern processors and deliver results which match those expected by a careful engineering analysis.

Time Domain Measurements

Measurements made directly in the time domain have the advantage that they are very simple to implement and provide definitive answers under all conditions. There is no assumption of a repetitive waveform and sudden changes in amplitude are handled correctly. There are two implementational complexities in a fixed sampling frequency approach: (1) In the equations defined below the simplest approach includes any DC component in the signal. However the effect of the DC component can be removed if it is significant. (2) If the frequency of the measured signal is not an exact multiple of the sampling frequency, then care must be taken to measure the actual frequency, use appropriate fractional points at the start and end of the integration, and perform normalization correctly.

Time Domain Calculations

$V_{rms} = \sqrt{\frac{1}{N}\sum_n V_n^2}$	RMS Voltage (V)
$I_{rms} = \sqrt{\frac{1}{N}\sum_n I_n^2}$	RMS Current (I)
$P = \frac{1}{N}\sum_n V_i I_i$	Active Power (Pa) – Calculation includes any DC component as well as all frequencies in the signal up to the Nyquist frequency.
$S = VA = V_{rms} I_{rms} = \sqrt{\frac{1}{N}\sum_{i=0}^{i=N-1} V_i^2 \bullet \frac{1}{N}\sum_{i=0}^{i=N-1} I_i^2}$	Apparent Power (Sa) - Calculation includes any DC component as well as all frequencies in the signal up to the Nyquist frequency.
$Q = \sqrt{S^2 - P^2}$	Reactive Power (Qa) – There is not a good formulation in the time domain for directly computing Q. We have adopted the approach of computing it from the “Power Triangle” assumption.

All summations must account for the exact number of samples in a cycle including fractional data points and properly normalize for the length of the cycle when doing multiple cycle summations to calculate energy related quantities.

Frequency Domain Measurements

An alternative approach to using the sampling data directly in the time domain is to use it to do a Fourier analysis on the waveforms. One of the basic assumptions of Fourier analysis is that the waveform is repetitive over the interval of analysis. While real world waveforms are not perfectly repetitive over long periods of time, the assumption of repetitiveness over a small number of cycles is usually quite good.

According to Fourier's Theorem any ***periodic*** signal can be represented in the following manner:

$$V(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n Cos(n\omega_0 t) + b_n Sin(n\omega_0 t)\right)$$

Frequency Domain Calculations

$V_{rms} = \frac{1}{\sqrt{2}}\left[\sum_n (a_{vn}^2 + b_{vn}^2)\right]^{1/2}$	RMS Voltage –[1][2]
$I_{rms} = \frac{1}{\sqrt{2}}\left[\sum_n (a_{in}^2 + b_{in}^2)\right]^{1/2}$	RMS Current –[1][2]
$V_1 = \frac{1}{\sqrt{2}}\left[a_{v1}^2 + b_{v1}^2\right]^{1/2}$	RMS Voltage –Fundamental Only[1]
$I_1 = \frac{1}{\sqrt{2}}\left[a_{i1}^2 + b_{i1}^2\right]^{1/2}$	RMS Current –Fundamental Only[1]
$P_t = \sum_n \left\lvert \vec{V}_n \bullet \vec{I}_n \right\rvert = \frac{1}{2}\sum_n (a_{vn}a_{in} + b_{in}b_{vn}) = \sum_n V_n I_n \cos(\theta_n)$	Active Power (Pt)– Active power computed by summing the vector dot products for each of the harmonics[1][2]
$P_1 = \left\lvert \vec{V}_1 \bullet \vec{I}_1 \right\rvert = \frac{1}{2}\left[a_{v1}a_{i1} + b_{v1}b_{i1}\right] = V_1 I_1 \cos(\theta_1)$	Active Power (P_1)– Active power for the fundamental frequency only.[1][2]
$S_t = \frac{1}{2}\left[\sum_n (a_{vn}^2 + b_{vn}^2)\sum_n (a_{in}^2 + b_{in}^2)\right]^{1/2}$	Apparent Power (St) – Apparent power computed by summing the Vrms times Irms for each harmonic.[1][2]
$S_1 = \frac{1}{2}(a_{v1}^2 + b_{v1}^2)^{1/2}(a_{i1}^2 + b_{i1}^2)^{1/2}$	Apparent Power (S_1) – Apparent power computed as Irms times Vrms for the fundamental only.[1]
$Q_t = \sum_n \left\lvert \vec{V}_n \times \vec{I}_n \right\rvert = \frac{1}{2}\sum_n (a_{vn}b_{in} - a_{in}b_{vn}) = \sum_n V_n I_n \sin(\theta_n)$	Reactive Power (Qt) – Reactive power computed by summing the vector dot products of each of the harmonics[1]
$Q_1 = \left\lvert \vec{V}_1 \times \vec{I}_1 \right\rvert = \frac{1}{2}\left(a_{v1}b_{i1} - a_{i1}b_{v1}\right) = V_1 I_1 \sin(\theta_1)$	Reactive Power (Q_1) - Reactive power for the fundamental only[1]

Notes:

(1) The a_0 component is generally not included but could be if desired.

(2) If N is sufficiently large so all frequencies in the signal are included and the signal is periodic, then this equation will give the exact same value as the same quantity measured in the time domain.

The above frequency domain formulations for Watts and VA generate numeric results for all periodic waveforms that are the same as those in the time domain. The formulation of the VAR calculation takes the single frequency definition of VAR

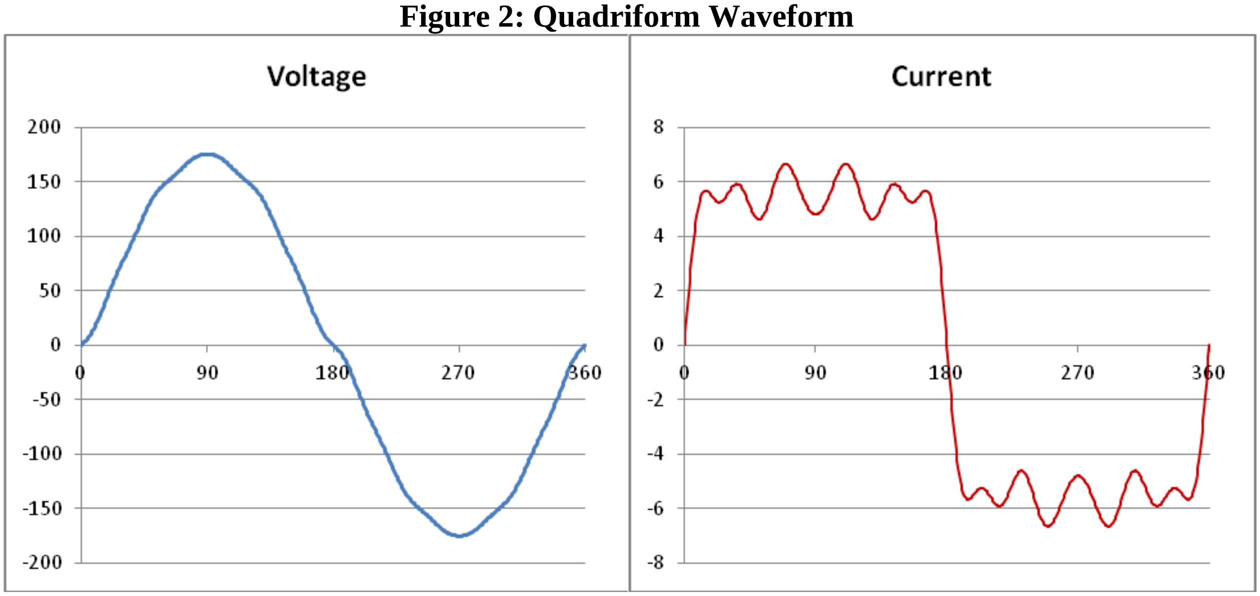
$$Q = VI\sin(\theta)$$

and extends it to the full Fourier spectrum. It should be noted that there are very large differences in the value for VARs between the two approaches. As an example consider one of several of the test waveforms currently being considered for harmonic testing in C12.20. The Quadriform waveform is found in IEC, OIML and C12 standards. The voltage and current waveforms are defined by:

Table 2 Quadriform Waveform

Harmonic	Voltage Amplitude % Vref	Phase	Current Amplitude % Iref	Phase	Energy
1	100	0	100	0	100.000
3	3.8	180	30	0	-1.140
5	2.4	180	18	0	-0.432
7	1.7	180	14	0	-0.238
11	1.1	180	9	0	-0.099
13	0.8	180	5	0	-0.040
			Total Energy		98.051(2)

Figure 2: Quadriform Waveform



Time Domain (Trapazoidal Integration)		Frequency Domain	
Vrms	**120.150**	**Vrms**	**120.150**
Irms	**5.36796**	**Irms**	**5.36796**
Active Power (Pt)	**588.306**	**Active Power (Pf)**	**588.306**
Reactive Power (Qt)	**264.323**	**Reactive Power (Qf)**	**0.000**
Apparent Power (St)	**644.958**	**Apparent Power (Sf)**	**644.958**

Note that with 512 samples per cycle and trapezoidal integration the answers for power and VA are identical using both methods: time domain and frequency domain. However, the VAR results are dramatically different. Since we can't calculate VAR directly in the time domain we applied the power triangle to the gross power and VA results. This yields a large number for VAR. This would indicate that there must be an "angle" between voltage and current. In reality for every harmonic the voltage and current are either in phase or 180 degrees out of phase. Under these conditions the VAR is zero. This is what the frequency domain calculation yields. There are no VARs if the voltage and current are in phase. This illustrates clearly why we have always made the caveat that "The power triangle only works for sinusoidal waveforms."